Chapter 4

Consumption and Saving

4.1 Introduction

Thus far, we have focussed primarily on what one might term *intratemporal* decisions and how such decisions determine the level of GDP and employment at any point in time. An intratemporal decision concerns the problem of allocating resources (like time) across different activities *within* a period. However, many (if not most) decisions have an *intertemporal* aspect to them. An intertemporal decision concerns the problem of allocating resources *across* time. For example, deciding how much to consume today can have implications for how much will be available to consume tomorrow. The decision of how much to invest must be made with a view as to how this current sacrifice is likely to pay off at some future date. If a government runs a deficit today, it must have in mind how the deficit is to be paid off in the future, and so on. Such decisions are inherently *dynamic* in nature. In order to understand how such decisions are made, we need to develop a dynamic model.

In this chapter, we focus on the consumption-savings choice of individuals. Since any act of saving serves to reduce consumption in the present and increase consumption in the future, the key decision involves how to optimally allocate consumption across time. We will study this choice problem within the context of a two-period model. The basic insights to be gleaned from a simple two-period model continue to hold true in a more realistic model that features many periods. In order to focus on the intertemporal aspect of decision-making, we abstract from intratemporal decisions. In particular, the working assumption here is that intratemporal decisions are independent of intertemporal decisions. This assumption is made primarily for simplicity and can be relaxed once the basic ideas presented here are well understood.

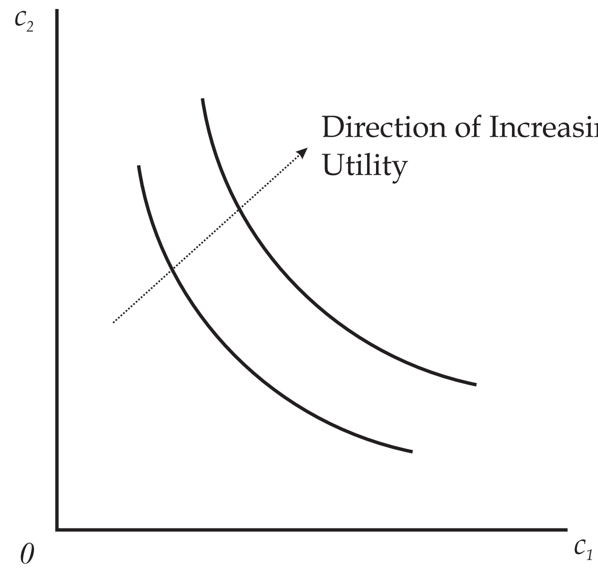
4.2 A Two-Period Endowment Economy

4.2.1 Preferences

Consider a model economy populated by a fixed number of individuals that live for two periods. These individuals have preferences defined over *time-dated* output (in the form of consumer goods and services). Let (c_1, c_2) denote an individual's lifetime consumption profile, where c_1 denotes 'current' consumption and c_2 denotes 'future' consumption. Note that consumption today is *not the same* as consumption tomorrow; *they are treated here as two distinct goods.* The assumption that people have preferences for time-dated consumption simply reflects the plausible notion that people care not only for their material well-being today, but what they expect in terms of their material well-being in the future. In what follows, we assume that there is no uncertainty over how the future evolves.

A lifetime consumption profile (c_1, c_2) can be thought of as a *commodity bundle*. The *commodity space* then is defined to be the space on non-negative commodity bundles and can be represented with a two-dimensional graph. We make the usual assumptions about preferences; i.e., more is preferred to less, transitivity, and convexity. We will also make the reasonable assumption that consumption at different dates are normal goods and that preferences can be represented with a utility function $u(c_1, c_2)$. Figure 4.1 depicts an individual's indifference curves in the commodity space.

FIGURE 4.1
Indifference Curves



4.2.2 Constraints

Individuals are endowed with an earnings stream (y_1, y_2). One can interpret y_j as the real per capita GDP in period $j = 1, 2$. From Chapter 2, we know that the level of GDP is determined by an intratemporal decision concerning the allocation of time to market-sector activities. Because we want to abstract from intratemporal decisions here, let us assume here for simplicity that these decisions are exogenously determined. Given these decisions, the individuals operate *as if* they are faced with an exogenous earnings profile (y_1, y_2). This assumption is reflected in our use of the label: an *endowment* economy.

The assumption that real per capita GDP is exogenous is not important for our purpose here. However, we make one more assumption that does turn out to be important; i.e., that output is non-storable. This is to say that output can not be held in the form inventory. Nor can it take the form of new capital goods, like business fixed investment. For this reason, it is perhaps best to think of output as taking the form of services and perishable goods. We will relax this assumption in a later chapter that discusses capital and investment.

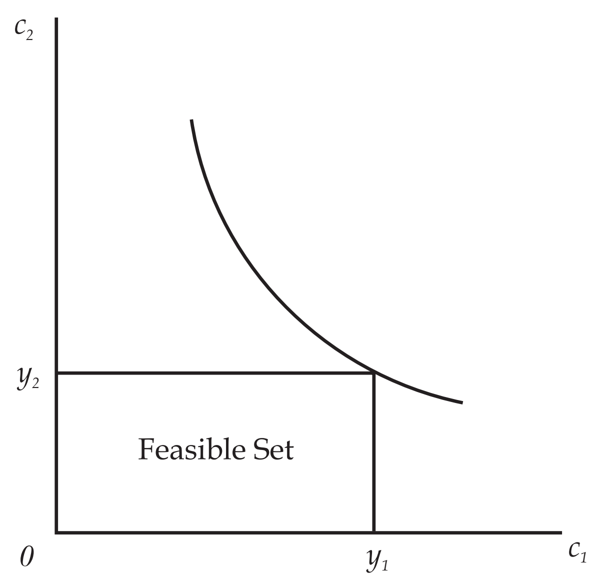
4.2.3 Robinson Crusoe

Before proceeding to develop the model further, it will be useful to pause a moment and to ask what the solution to the individual's choice problem would look like if that individual was to operate in the world just described above. In particular, assume that the individual is living alone on an island (like Robinson Crusoe, in Dafoe's famous novel). Our Robinson Crusoe is endowed with y_1 units of output (e.g., coconuts) today and expects y_2 units of output to be available in the future (coconuts that are due to mature on his trees in the next period). Remember that we are assuming that coconuts are not storable. Mathematically, the choice problem is given by:

Choose (c_1, c_2) to maximize $u(c_1, c_2)$ subject to: $c_1 \leq y_1$ and $c_2 \leq y_2$.

The solution to this choice problem is trivial: Choose $c_1^D = y_1$ and $c_2^D = y_2$. In other words, the best that Crusoe can do is to simply consume his entire income in each period. This solution is depicted diagrammatically in Figure 4.2.

FIGURE 4.2
Robinson Crusoe



4.2.4 Introducing a Financial Market

Our Robinson Crusoe economy is a *metaphor* for an environment in which individuals do not have access to markets (in this case, financial markets). In reality, individuals do have access to financial markets. A financial market is a market on which *claims* to time-dated consumption can be traded. These claims represent *promises* to deliver output at specified dates. In what follows, we assume that promises of this form can be enforced at zero cost. We implicitly made the same assumption in Chapter 2, where we thought of individuals supplying labor in exchange for claims against output; these claims were then redeemed at the firm that issued them at zero cost (i.e., there was no issue of a firm reneging on its promises). In the present context, the fact that financial claims can be costlessly enforced implies that there is no risk of default.

There are two goods in this model economy (current and future consumption) and hence there can only be one market and one price. On this market, people exchange claims for time-dated consumption and the rate at which these claims exchange measures the (relative) price of these two goods. Let R denote the price of current consumption measured in units of future consumption. That is, given some market-determined price R, an individual is able to borrow or lend one unit of c_1 in exchange for R units of c_2. For this reason, R is called the (gross) *real* rate of interest.[1] Note that the act of borrowing or lending current consumption in a private debt market is equivalent to selling or buying claims to future consumption.

- **Exercise 4.1.** You ask your buddy to buy you a beer tonight; in exchange, you promise to buy him a beer tomorrow night. What is the implicit real rate of interest in this exchange? Explain how borrowing the one beer tonight is equivalent to selling a claim against future beer.

In what follows, we shall simply assume that there is a market for risk-free private debt with an exogenously determined real interest rate R. Later on, we will discuss the economic forces that determine R, but for now we just take R as part of the environment.

If individuals have access to a financial market, then they will be faced with an *intertemporal budget constraint* (IBC). To derive this constraint, we begin by defining the concept of saving. Saving can be defined in general terms as current income minus expenditures on current needs. In the present context, saving is given by:

$$s \equiv y_1 - c_1. \tag{4.1}$$

Notice that, generally speaking, saving can be either positive or negative. If $s > 0$, then the individual is saving (purchasing bonds); if $s < 0$, then the

[1] The real interest rate is not to be confused with the nominal interest rate. The former represents the relative price of time-dated *output*, while the latter represents the relative price of time-dated *money* (a topic that will be discussed shortly).

individual is borrowing (selling bonds). Note that a 'bond' in the present context refers to a privately-issued liability (i.e., a personal promise to deliver stuff in the future).

- **Exercise 4.2.** When you approach the bank for a loan, you are in effect offering to sell the bank a bond. True, False, Uncertain and Explain.

If individuals can be expected to make good on their promises, then the second-period budget constraint is given by:

$$c_2 = y_2 + Rs. \tag{4.2}$$

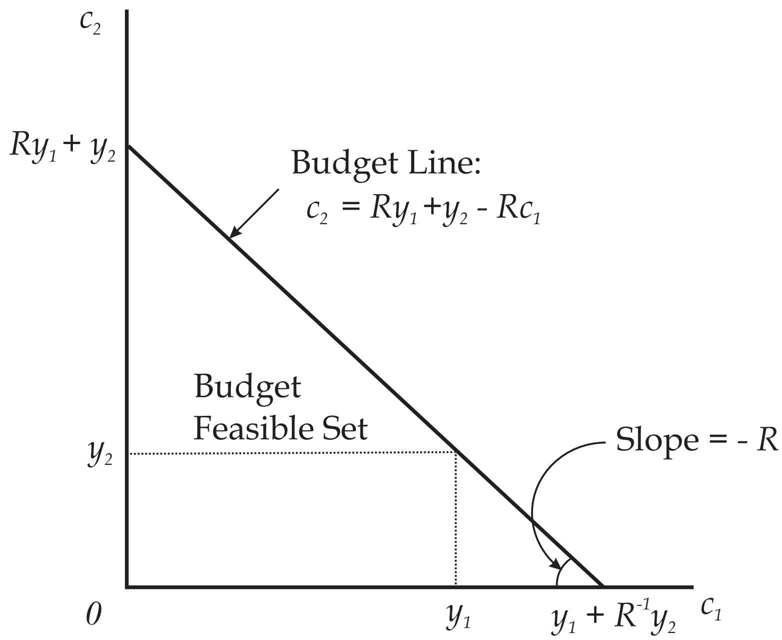
What this constraint tells us is that the individual's future consumption spending cannot exceed the sum of his earnings *plus* the interest and principal on any saving. Note that if $s < 0$, then Rs represents the amount of output that the individual is obliged to repay his creditors. Also note that since R is the *gross* interest rate, the *net* interest rate is given by $r = (R - 1)$; in other words, $(1 + r) = R$. The quantity rs is called 'interest income' if $s > 0$ and is called the 'interest charges' if $s < 0$.

Substituting the definition of saving (4.1) into the second-period budget constraint (4.2) yields an equation for the *budget line*:

$$c_2 = Ry_1 + y_2 - Rc_1. \tag{4.3}$$

This budget line tells us which combinations of (c_1, c_2) are *budget feasible*, given an endowment (y_1, y_2) and a prevailing interest rate R. Notice that the budget line is a linear function, with a slope equal to $-R$. This budget line is graphed in Figure 4.3.

FIGURE4.3
Intertemporal Budget Constraint



One can rearrange the budget line (4.3) in the following useful way:

$$c_1 + \frac{c_2}{R} = y_1 + \frac{y_2}{R}. \tag{4.4}$$

The right-hand-side of the equation above is the *present value* of the individual's lifetime earnings stream. This is just a measure of *wealth* measured in units of current consumption (and is represented as the x-intercept in Figure 4.3). We can also measure wealth in units of future consumption; i.e., $Ry_1 + y_2$. This is called the *future value* of an individual's lifetime earnings stream (and is depicted by the y-intercept in Figure 4.3). Likewise, the left-hand-side of equation (4.4) measures the present value of the individual's lifetime consumption spending. Hence, the intertemporal budget constraint tells us that the present value of lifetime consumption spending cannot exceed one's wealth. This constraint puts an upper bound on the amount that an individual can borrow.

- **Exercise 4.3.** Consider an individual with an endowment of beer given by $(y_1, y_2) = (0, 12)$. That is, the individual has no beer today, but is expecting a shipment of beer tomorrow. If the (overnight) real rate of interest is $R = 1.20$ (a 20% net interest rate), what is the maximum amount of beer that this person can borrow today?

From the intertemporal budget constraint (4.4), we see that $c_1 = y_1$ and $c_2 = y_2$ is budget feasible. Therefore, we can conclude that the IBC *always* passes through the endowment point. But unlike the case of Robinson Crusoe, we see that an individual with access to a financial market is much less constrained in how he can allocate his consumption over time. In particular, $c_1 < y_1$ and $c_2 > y_2$ is possible (by saving), or $c_1 > y_1$ and $c_2 < y_2$ is possible (by borrowing).

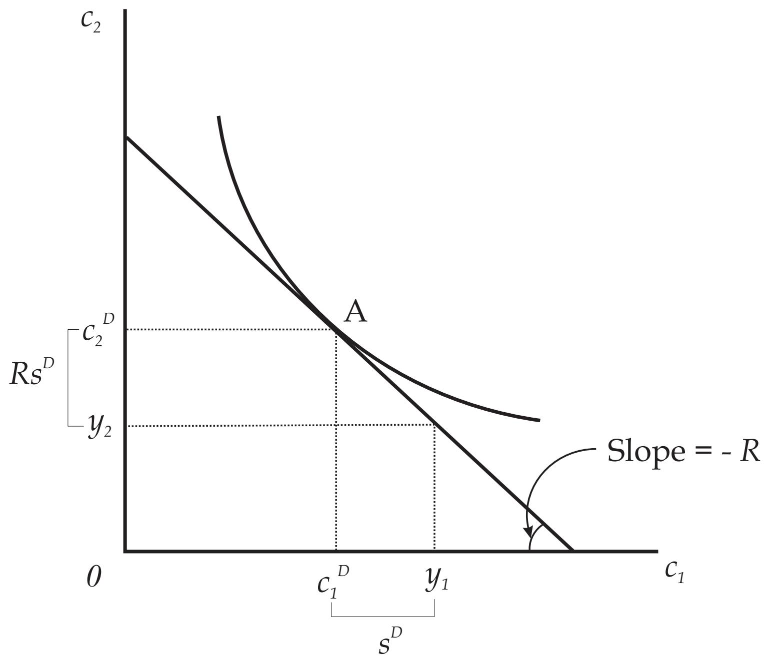
4.2.5 Individual Choice with Access to a Financial Market

For an individual in possession of an endowment (y_1, y_2) and who is able to borrow or lend at the interest rate R, the choice problem can be stated mathematically as follows:

Choose (c_1, c_2) to maximize $u(c_1, c_2)$ subject to: $c_1 + \frac{c_2}{R} = y_1 + \frac{y_2}{R}$.

The solution to this choice problem is a pair of demand functions (c_1^D, c_2^D) that depend on the parameters that describe the person's physical and economic environment; i.e., y_1, y_2, R and u. Once c_1^D is known, one can calculate the person's desired saving function s^D from the definition of saving (4.1); i.e., $s^D = y_1 - c_1^D$. The solution is depicted graphically in Figure 4.4 as point A.

FIGURE 4.4
Consumption - Saving Choice



There are two mathematical conditions that completely describe point A in Figure 4.4. First, observe that at point A, the slope of the indifference curve is equal to the slope of the budget line. Second, observe that point A lies on the budget line. In other words,

$$MRS(c_1^D, c_2^D) = R; \tag{4.5}$$
$$c_1^D + \frac{c_2^D}{R} = y_1 + \frac{y_2}{R}.$$

- **Exercise 4.4.** Identify the exogenous and endogenous variables of the theory developed above. What sort of questions can this theory help us answer?

- **Exercise 4.5.** Suppose that the utility function takes the following form: $u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$, where $\beta \geq 0$ is a preference parameter. Explain how the parameter β can be interpreted as a 'patience' parameter. In particular, what would β be equal to for an individual who 'doesn't care' about the future?

- **Exercise 4.6.** Suppose that preferences are such that $MRS = c_2/(\beta c_1)$. Use (4.5) to derive the consumer demand function c_1^D. How does c_1^D depend on β? Explain.